MICROLENSING: CURRENT RESULTS AND FUTURE PROSPECTS

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Abstract. The initial results of microlensing surveys toward the Galactic bulge and the LMC are puzzling. Toward the LMC, the total mass in MACHOs is of order half that required to explain the dark matter, but the estimated MACHO mass ($\sim 0.4\,M_\odot$) is most consistent with stars that should have been seen. Toward the bulge, the total mass is consistent with that given by dynamical estimates of the bulge mass, but the observed timescales seem to require that a large fraction of the bulge is made of brown dwarfs. I discuss possible experiments to resolve these puzzles. I also discuss several applications of microlensing to non-dark-matter problems including searching for planets and imaging the black hole at the center of a quasar with 10^{-10} arcsec resolution.

1. Introduction

Gravity bends light. Any mass concentration therefore acts as a gravitational lens. (See Schneider, Ehlers, & Falco 1992 for a review.) The magnification and distortion induced by this (or any) lens can be described as a matrix A_{ij}, the transformation from the source plane to the image plane. By Liouville's (1837) theorem, surface brightness is conserved, so the magnification is given by the determinant of this transformation, $A = |A_{ij}|$.

Microlensing is, by definition, gravitational lensing where the image structure is not resolved. (See Paczyński 1996; Roulet & Mollerach 1997; and Gould 1996 for reviews.) The only observable is therefore the total magnification from all the images. The most important case is microlensing of a point source by a point mass. If the lens were perfectly aligned with

the observer-source line of sight, the source would be imaged into a ring of angular radius, θ_e, called the Einstein radius

$$\theta_e = \sqrt{\frac{4GM}{Dc^2}}; \qquad D \equiv \frac{d_{\mathrm{ol}}d_{\mathrm{os}}}{d_{\mathrm{ls}}}, \qquad (1)$$

where M is the mass of the lens, and d_{ol}, d_{ls}, and d_{os} are the distances between the observer, lens, and source. If the alignment is not perfect, there are two images, one inside and one outside the Einstein ring and with magnifications

$$A_{\pm} = \frac{A \pm 1}{2}; \qquad A(x) = \frac{x^2 + 2}{x\sqrt{x^2 + 4}}, \qquad (2)$$

where x is the projected separation of the source and lens in units of the Einstein ring.

The signature of microlensing is a specific form of time variability of the source flux. If the lens moves with uniform transverse velocity $\mathbf{v}$ relative to the observer-source line of sight, then the separation x is given by the Pythagorean theorem,

$$x(t) = \sqrt{\frac{(t - t_0)^2}{t_e^2} + \beta^2}, \qquad (3)$$

where β is the impact parameter in units of θ_e, t_0 is the time of maximum magnification, and t_e is the Einstein crossing time,

$$t_e = \frac{d_{\mathrm{ol}}\theta_e}{v}. \qquad (4)$$

Both the power and limitations of microlensing are summarized in these four equations. On the one hand, the microlensing light curve is described by only three parameters, t_0, β, and t_e. Thus microlensing can be distinguished from other forms of stellar variability which are about 1000 times more common. On the other hand, all of the information about the lens is contained in t_e which is itself a complicated combination of M, d_{ol}, and v.

Here I will summarize some of the major results obtained from microlensing experiments to date and also discuss some future possibilities.

2. Dark Matter and Dim Matter

Paczyński (1986) proposed that one could search for Machos (lumps of dark matter) in the halo of the Milky Way by monitoring stars in the Large Magellanic Cloud (LMC) for microlensing events. Astronomers reacted to

this the same way they usually do whenever someone comes up with a good new idea: they said it was impossible. The problem is that the probability that any given star is microlensed at any given time is $< 10^{-6}$, so that it would be necessary to monitor millions of stars to get a few events. This seemed to be beyond the capabilities of individual observers. However, particle physicists were also interested in this problem because if the dark matter is not made of Machos, it may be new fundamental particles. The experimental requirements are modest by particle physics standards. Two groups (MACHO, Alcock et al. 1997b; EROS, Ansari et al. 1996), each composed of particle physicists and astronomers, initiated Macho searches toward the LMC.

The initial results of these LMC searches are extremely puzzling. MACHO finds a total of 8 candidate events from their first two years of data with time scales t_e ranging from about 2.5 weeks to 2.5 months. While one of these events is most likely due to a stellar lens in the disk of the Milky Way (Gould, Bahcall, & Flynn 1997), one is likely due to a binary-star lens in the LMC, and one is quite possibly a variable star (and thus not microlensing), the remaining 5 events are difficult to explain within the context of standard models of our Galaxy. If they are due to dark objects in the halo, these objects seem to account for $\sim 50\%$ of the dark matter. The problem with this interpretation lies in the time scales. From the microlensing equations of § 1, it is clear that $t_e \propto \sqrt{M}$ and so contains some information about mass, but that it also depends of $d_{\rm ol}$ and v. The distributions of these latter quantities is fixed for any given halo model, so for a given mass, there is a probability distribution of time scales. The best estimate for the mean Macho mass is $\sim 0.4 \pm 0.2 \, M_\odot$. What could such objects be? They cannot be made of primordial material (H and He) because they would be M stars and easily visible. In fact star counts show that halo M stars are about 100 times less numerous than is required to solve the dark matter problem. That is why there is a dark matter problem. The estimated Macho mass is consistent with white dwarf (WD) masses, but several arguments make this solution seem implausible. Star counts place a 95% confidence upper limit on the density of WDs that is close to the density required to explain the Macho events (Flynn, Gould, & Bahcall 1996). Moreover, WD progenitors would pollute the halo with metals and would 'light up' distant galaxies with red-giant light if this were indeed the universal explanation of dark matter (and not just in our Galaxy). It is a mark of the paucity of other plausible ideas to explain the observed microlensing events that the WD scenario is taken seriously at all.

An interesting set of alternative ideas is that the majority of the events do not arise from the halo, but rather from stellar objects in the LMC itself (Sahu 1994) or a thick disk in the Milky Way (Gould 1994a). Unfortunately,

dynamical constraints appear to limit the contributions of both structures to levels that are well below the observed lensing rate (Gould 1995b). In brief, there are no sensible ideas to account for the observed lensing toward the LMC.

At first sight, the situation appears quite different toward the Galactic bulge, where four groups have carried out lensing observations (OGLE, Udalski et al. 1994; MACHO, Alcock et al. 1997a; DUO, Alard 1996; EROS). Since the line of sight passes through the disk and the bulge itself, many events are expected from ordinary stars. Many are seen and the time scales are broadly consistent with stellar masses. Zhao, Spergel, & Rich (1995) showed that the time scale distribution could be explained if the dynamically measured mass of the bulge ($\sim 2 \times 10^{10} \, M_\odot$) were distributed in a Salpeter (power-law) mass function between 0.6 and $0.08 \, M_\odot$, the latter value being the hydrogen-burning limit. Unfortunately, this simple picture does not hold up under closer examination. First, all of the bulge mass cannot be in objects $M < 0.6 \, M_\odot$, since the bulge luminosity function (LF) has been measured for $M > 0.5 \, M_\odot$. These account for about half of the bulge mass but very few of the observed lensing events (Han 1997). If the bulge LF (Light, Baum, & Holtzman 1997) is extended according to the locally measured disk LF (Gould, Bahcall, & Flynn 1996), then about 2/3 of the bulge mass is accounted for, but hardly any of the short ($t_e \sim 10 \, \mathrm{day}$) lensing events. Only if the last 1/3 of the bulge mass is assumed to be in brown dwarfs can the bulge lensing observations be explained (Han 1997).

In brief, both the LMC and bulge lensing observations are difficult to explain, but for opposite reasons. The LMC events seem to require lenses that are so massive ($\sim 0.4 \, M_\odot$) that they should shine and be noticed. The bulge events seem to require a new population of substellar objects not previously detected.

3.   Resolving the Macho Mysteries

Clearly, the best way to figure out what these objects are is to determine their individual masses, velocities, and distances. To date this has not been possible because the only information available is the time scale, t_e, which is a complicated combination all three: $t_e = t_e(M, d_{\mathrm{ol}}, v)$. Two additional pieces of information are needed to fully break this degeneracy. One parameter that one might hope to measure is the size of the Einstein ring projected onto the source plane. Another is the Einstein ring projected onto the plane of the observer. These are respectively,

$$\hat{r}_e \equiv d_{\mathrm{os}}\theta_e; \qquad \tilde{r}_e \equiv D\theta_e. \tag{5}$$

(Note that, since d_{os} is generally known reasonably well, determining $\hat{r}_e$ is equivalent to determining θ_e.) In either case, there must be some standard

ruler in the source plane or in the observer plane and there must be some effect that depends on the size of the Einstein ring relative to that ruler. If both parameters were measured, then one could determine M, d_{ol}, and v. For example,

$$M = \frac{c^2}{4G}\tilde{r}_e\theta_e. \qquad (6)$$

Even if only one of these two quantities were measured for a large sample of events, the character of the events would be substantially clarified. For example, if $\tilde{r}_e$ were measured, then one would also know the "projected speed",

$$\tilde{v} = \frac{\tilde{r}_e}{t_e} = \frac{d_{\mathrm{os}}}{d_{\mathrm{ls}}}v. \qquad (7)$$

This quantity is $\sim 50\,\mathrm{km\,s^{-1}}$ for disk lenses, $\sim 300\,\mathrm{km\,s^{-1}}$ for halo lenses, and $\sim 2000\,\mathrm{km\,s^{-1}}$ for LMC lenses. Hence these populations could be easily separated.

Where is one to find these standard rulers? By far, the best plan would be to create such a ruler in the plane of the observer by launching a parallax satellite into solar orbit (Refsdal 1966; Gould 1994b, 1995a; Boutreux & Gould 1996; Gaudi & Gould 1997a). Since $\tilde{r}_e \sim \mathcal{O}(\mathrm{AU})$, there is a significant fractional vector displacement in the Einstein ring of the event as seen from the satellite relative to the Earth, $\boldsymbol{\Delta x} = \mathbf{d}_{\mathrm{sat}}/\tilde{r}_e$, where $\mathbf{d}_{\mathrm{sat}}$ is the position of the satellite relative to the Earth. Hence, by measuring the difference in impact parameters $\Delta\beta = \beta' - \beta$ and difference in times of maximum $\delta t = t'_0 - t_0$ between the event as seen from the satellite and from the Earth, one can determine $\boldsymbol{\Delta x} = (\Delta t/t_e, \Delta\beta)$ and so $\tilde{r}_e$. Another method to measure the same quantity is to use the Earth's orbit as baseline (Gould 1992b). Unfortunately, most events end before the Earth has moved far enough to generate a significant effect. Nevertheless, $\tilde{r}_e$ has been measured for one event using this method (Alcock 1997a).

The most ubiquitous standard ruler in the source plane is the source itself whose angular radius θ_* is known from its color, magnitude, and Stefan's Law. If the lens transits the source (at say, $x = x_*$), the light curve will deviate from its standard form and one can therefore measure $\theta_e = \theta_*/x_*$ (Gould 1994a; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994). Unfortunately, the fraction of events for which this is possible is only $\sim \theta_*/\theta_e$, i.e., $< 5\%$ for the bulge and $< 1\%$ for the LMC. For bulge events, there are a variety of other methods to measure θ_e, notably optical/infrared photometry (Gould & Welch 1996) and infrared interferometry (Gould 1996). For the LMC, however, there are only two methods known that could plausibly provide information about θ_e. First, if the source happens to be a binary, the separation between the stars can be used as a standard ruler which is enormously larger than the physical extent of the individual stars (Han

& Gould 1997). Second, lensing of rapidly rotating sources (like A stars) creates an apparent line shift because the redshifted side of the star is magnified by a different amount than the blueshifted side. This allows one to measure θ_*/θ_e even if the impact parameter is many source radii (Maoz & Gould 1994).

In general, the various techniques discussed in this section require significant investments in observational effort and/or money. However, the methods are practical and well within present capabilities. It is possible to figure out what the lenses are if we make the effort.

4. Pixel Lensing of M87

For the remainder of the presentation, I will focus on what the future of microlensing may look like. This is highly speculative, but for such a new and rapidly developing field, rampant speculation is quite in order ... and may even prove productive. I begin by discussing possible microlensing of M87, the central galaxy in the Virgo cluster. This raises two immediate questions: how is it possible to observe microlensing events in a galaxy whose stars are completely unresolved, and why bother to observe them anyway?

I will not spend much time discussing how microlensing of unresolved stars can be observed. Most people even in the microlensing business considered that it was impossible when it was first proposed by Crotts (1992) and Baillon et al. (1993). But both groups have now demonstrated its feasibility in observations toward M31 (Tomaney & Crotts 1996; Ansari et al. 1997). Crotts will describe this success immediately following my presentation.

Rather let me focus on why M87 is an especially interesting target. The microlensing results from MACHO and EROS indicate that perhaps half of the dark matter in the Milky Way halo is in Machos. That is, the total mass of Machos is equal to or perhaps twice as large as the mass of all the stars in the known components of the Galaxy, the bulge and the disk. Hence, one might suppose that as the Galaxy was forming and was still roughly spherical, half of the available gas was processed into Machos. The remaining gas collapsed into a proto-disk and proto-bulge which went on to form the visible Galaxy we know and love. Imagine then a Milky-Way like galaxy forming on the outskirts of the Virgo cluster. Like the real Milky Way, it would process half of its gas into Machos with the remaining gas beginning to collapse into a proto-bulge and proto-disk. But before these collapsed gas clouds could form many stars, the galaxy would fall through the center of the cluster and would be stripped of its gas by the hot intracluster medium. The galaxy would become a dark Macho galaxy. The total mass in Machos would be about equal to the mass in gas,

i.e., 20% or so of the total mass of the cluster. The Macho galaxy might remain intact or dissolve, but in either event its Machos would give rise to microlensing events of M87. Detection of these events from the ground is probably not possible, but 10 continuous days of observations by the Hubble Space Telescope (HST) would yield ~ 30 events and so test this scenario directly (Gould 1995e).

There are many other applications of microlensing outside the Local Group, such as probing the star formation history of the universe (Gould 1995d) and measuring the transverse velocities of distant galaxies (Gould 1995c). However, time is short so I move on to an application closer to home.

5. Planet Detection

Microlensing can be used to detect anything that is dark. One interesting possibility is planets (Mao & Paczyński 1991; Gould & Loeb 1992). Suppose that a star is being microlensed by another star. Such events happen frequently toward the bulge. The light from the source star comes to us along two paths, one on either side of the lensing star. If the lensing star has a planet, and one of the light trajectories happens to come near that planet, then the planet will further deflect the light causing a deviation of the light curve from the standard form discussed in § 1. The deviation will be shorter than the event as a whole by a factor $\sqrt{m_p/M}$ where m_p is the mass of the planet. That is, the deviation will most likely last less than a day for a Jupiter-mass planet or smaller. It might therefore be missed by the ordinary microlensing search observations since these are typically carried out only once per day. However, the size of the deviation will typically be large, so that if the deviation is observed repeatedly, there will be no question that a planet has been detected. Hence, one should attempt to organize round-the-clock (i.e., round-the-world) observations once every few hours to catch such events. Two groups have begun such follow-up observations using observatories in Chile, South Africa, Israel, Australia, and New Zealand (Albrow et al. 1996; Pratt et al. 1996). Substantial improvements in these observations are expected when two optical/infrared cameras are placed on near-dedicated telescopes to join this follow-up program (D. DePoy 1997, private communication). The theoretical problems associated with the analysis of planetary light curves initially seemed rather daunting because the planetary Einstein ring is generally of the same size as the source star. However, substantial progress is now also being made on this front as well (Bennett & Rhie 1996; Gaucherel & Gould 1997; Gaudi & Gould 1997b).

6. Femtolens Imaging of Quasar Black Holes

Microlensing is developing with incredible speed. One indication of this is that while most of the ideas discussed in the previous sections were considered "crackpot" (or more politely, "too advanced for their time") when they were first proposed, many led almost immediately to new observational programs and the detection of new effects. Paczyński's (1986) original microlensing proposal is the most famous example of this, but there are many others. I already discussed the rapid implementation of the Crotts (1992) and Baillon et al. (1993) idea for pixel lensing. Finite source effects and ground-based parallax were both observed within 2 years of first being predicted. The proposal to search for planets was taken up by two world-wide collaborations within 3 years. The idea for a parallax satellite became a NASA proposal and pixel lensing of M87 became an HST proposal, both within 1 year (although neither is yet successful). If the most outrageous ideas that theorists can invent come to pass within a couple of years, then certainly we are not being imaginative enough! Here I present an attempt to overcome this shortcoming: femtolens interferometry of quasar black holes.

To explain femtolens interferometry, I must first describe simple femtolensing. Recall from § 1 that for a simple point-mass lens, there are two images. When I calculated the total magnification of the lens, I simply added the two magnifications together, $A = A_+ + A_-$. However, if the point source is truly a point, then the two images will arrive separated by a time delay Δt. To a good approximation

$$\Delta t(x) \simeq \frac{8GM}{c^3}x. \tag{8}$$

Hence, for light at wavelength λ, there will also be a phase delay $\phi = c\Delta t/\lambda$, and the true magnification will be

$$A = \mathcal{A}_+ \cos^2 \frac{\phi}{2} + \mathcal{A}_- \sin^2 \frac{\phi}{2}; \qquad \mathcal{A}_\pm = (A_+^{1/2} \pm A_-^{1/2})^2 = \left(1 + \frac{4}{x^2}\right)^{\pm 1/2}. \tag{9}$$

Normally interference is not important because real sources are so big that interference effects at different points on the source have different phases which cancel one another out. However, if γ-ray bursts come from cosmological distances, and if they were lensed by asteroid-mass objects, their spectra would show oscillations with peak-to-trough variations of $\mathcal{A}_+/\mathcal{A}_- = (1 + 4/x^2)$, which would easily be noticed (Gould 1992a). Thus, γ-ray bursts could be used to probe for or put limits on such objects.

Femtolens interference can be extended to femtolens interferometry, but some additional investment is required (Gould & Gaudi 1997). First one must find a nearby ($\lesssim 30\,\mathrm{pc}$) dwarf star that is perfectly aligned with a

distant quasar. The star is to serve as the "primary lens" of a giant telescope to image the quasar. Unfortunately, even if such an alignment happened to occur, the transverse motion of the dwarf ($\sim 40\,\mathrm{km\,s^{-1}}$) would wreck the telescope as soon as it was set up. So it will be necessary to use a satellite to bring the "secondary optics" of the telescope into alignment with the dwarf-quasar line of sight ... and keep it there. There should be such a point of alignment within $\sim 45\,\mathrm{AU}$ of the Sun. If the dwarf star were isolated, the quasar would be imaged into two images. However, most dwarfs have binary companions. Such a companion is just what is needed to create a femtolens imaging telescope. It creates an "astigmatism" in the lens called a "caustic". If the quasar lies inside the caustic, then there are 5 images (instead of two for a point lens). One of these images is close to companion and will be ignored. If the quasar lies close to a cusp of the caustic, then three of the remaining images will be very highly magnified and lie on one side of the dwarf, while the fourth image will be only moderately magnified and lie on the opposite side of the dwarf. It will be ignored. The typical magnifications of the three images are $\sim 10^6$ in one direction, but there is an actual demagnification by a factor of 2 in the other direction. That is, each image will be highly elongated: it can be resolved in one direction, but not the other. For example, if a quasar black hole has a mass $M \sim 10^8\,M_\odot$, then its Schwarzschild radius is $\sim 1\,\mathrm{AU}$. At a cosmological distance it therefore subtends 10^{-9} arcsec. Its image will then be 1 mas $\times 10^{-6}$ mas. The first dimension is easily resolved with a space-based telescope. The second is not. The point of femtolens interferometry is to resolve the second dimension.

If the image of the quasar is resolved in one dimension, then light from different portions of the image can be brought together and analyzed in a spectrograph. Each portion will contain light from a one-dimensional strip through the quasar. These strips generally intersect one another only in a limited region. If the two portions are brought together, then only the light from this limited region suffers interference. Actually, each such region contains subregions with different relative time delays between the two image portions. The interference pattern is the Fourier transform of the this time-delay structure. It can reveal structure as small as $\sim 1/10$ AU, which is the separation in the source plane at which the relative time delay differs by one λ/c where λ is the typical wavelength of optical light.

Of course, there are a few engineering problems associated with this idea. It is easy to get a satellite to 45 AU, but this one must be given an additional boost of $\sim 40\,\mathrm{km\,s^{-1}}$ once it gets there. The mirror system must extend about 350 m in a one-dimensional array in order to re-image the gravitationally lensed quasar images. This is not out of line with other plans for space-based interferometers. However, in this case, the mirror system must be accelerated by $\sim 20\,\mathrm{cm\,s^{-2}}$ about once every 10 hours in order

to counter the Sun's gravity, and the mirror system must restablize after each such jolt. However, microlensing has met previous challenges and I am confident it will meet these as well.

7. Conclusion

More good microlensing ideas are needed.

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